ABSTRACT ALGEBRA II

MATH 4120/8126

Course Description:

An introduction to ring and field theory. Various classes of commutative rings are considered including polynomial rings, and the Gaussian integers. Examples of fields include finite fields and various extensions of the rational numbers. Concepts such as that of an ideal, integral domain, characteristic and extension field are studied. The course culminates with an introduction to Galois theory. Applications include the resolution of two classical problems: the impossibility of angle-trisection and the general insolvability of polynomial equations of degree 5 or higher. **3 credits**

Prerequisites:

Undergraduate and Graduate: MATH 4110/8116 with a C- or better or permission of instructor

Overview of Content and Purpose of the Course:

This second course in abstract algebra covers commutative rings, ideals, integral domains, fields, and some basic linear algebra. The division algorithm and unique factorization are discussed in connection with the Gaussian integers, some basic Algebraic Number Theory, and Fermat's Last Theorem. Polynomial rings, the factorization of polynomials lead to a study of finite fields, and extensions of field, including extensions of the rational numbers. This course culminates with an introduction to Galois theory, which allows us to tie the solvability of a polynomial equation to group theory. As a result, we are able to apply the most advanced material from both abstract algebra courses to solve one old classical problem by proving the general insolvability of a polynomial equation of degree 5 or higher. Applications also include the impossibility of angle-trisection.

Major Topics:

1) Introduction to Rings
   a. Examples of Rings
   b. Properties of Rings
   c. Subrings

2) Integral Domains
   a. Definition and Examples
   b. Fields
   c. Characteristic of a Ring

3) Ideals and Factor Rings
   a. Ideals
   b. Factor Rings
   c. Prime Ideals and Maximal Ideals

4) Ring Homomorphisms
   a. Definition and Examples
   b. Properties of Ring Homomorphisms

5) Polynomial Rings
   a. Notation and Terminology
   b. The Division Algorithm and Consequences

6) Factorization of Polynomials
   a. Reducibility Tests
   b. Irreducibility Tests
   c. Unique Factorization in Z[x]

7) Divisibility in Integral Domains
   a. Irreducibles, Primes
   b. Historical Discussion of Fermat's Last Theorem
   c. Unique Factorization Domains
   d. Euclidean Domains

8) Vector Spaces
   a. Definition and Examples
   b. Subspaces
   c. Linear Independence

9) Extension Fields
   a. The Fundamental Theorem of Field Theory
   b. Splitting Fields
   c. Zeros of an Irreducible Polynomial

10) Algebraic Extensions
   a. Characterization of Extensions
   b. Finite Extensions
   c. Properties of Algebraic Extensions

11) Finite Fields
   a. Classification of Finite Fields
   b. Structure of Finite Fields
   c. Subfields of a Finite Field

12) Geometric Constructions
   a. Historical Discussion of Geometric Construction
   b. Constructible Numbers
   c. Angle-Trisectors and Circle-Squares

13) An Introduction to Galois Theory
  a. Fundamental Theorem of Galois Theory
  b. Solvability of Polynomials by Radicals
  c. Insolvability of a Quintic

14) Cyclotomic Extensions
  a. Motivation
  b. Cyclotomic Polynomials
  c. The Constructible Regular *n*-gons

<u>Textbook</u>:

Gallian, Joseph A. *Contemporary Abstract Algebra*, *7th ed*. Belmont: Brooks/Cole, 2009.

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